Concentration Polarization in a Narrow Reverse Osmosis Membrane Channel

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Concentration polarization in a narrow reverse osmosis channel is bounded by the channel height and under the influence of variable transverse velocity. An attempt was made in this article to quantify concentration polarization in such a narrow membrane channel. The transverse velocity in the membrane channel was first determined and its impact on concentration polarization was investigated. Based on the concept of retained salt, analytical equations were developed for the wall salt concentration at an arbitrary point in the narrow membrane channel. Finally, development of concentration polarization in typical reverse osmosis channels under various conditions was numerically simulated and discussed. Interesting results on the details of concentration polarization in the narrow reverse osmosis channel that had never been reported before were revealed with this mechanistic model. © 2009 American Institute of Chemical Engineers *AIChE J*, 56: 143–149, 2010*
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Introduction

Concentration polarization is an inherent phenomenon of reverse osmosis (RO) processes that a higher salt concentration occurs on the membrane surface than that in the bulk solution as a result of salt retention by the RO membranes.[1-3] The development of concentration polarization in a membrane channel is a complex phenomenon that is affected by the geometry and flow conditions of the channel. Most of models available for concentration polarization were developed with the assumption that the concentration polarization layer is much smaller than the geometrical height of the membrane channel.[4-9] With this assumption, concentration polarization can be treated as to develop unboundedly in a semiplane over the membrane surface. In addition, the transverse velocity in the concentration polarization layer can be approximated by a constant that equals to the permeate velocity through the membrane.

RO processes in water desalination and purification are characterized with narrow channels that are typically a fraction of 1 mm in height and several meters in length.[10-13] The recovery of a membrane channel is usually 40% or more for seawater desalination and even higher for brackish water desalination. As a result, the salt concentration in the retentate stream is substantially higher than that in the feed water. In addition, the existence of spacers in the RO channel will help the expansion of the concentration polarization layer in the channel height. In such cases, the assumption that concentration polarization develops in a boundary layer much smaller than the channel height may become invalid except for a short segment from the entrance of the membrane channels. In other words, the models for concentration polarization based on this assumption may not be applicable to the narrow RO channels.

An attempt was made in this article to quantify concentration polarization in the narrow RO channels, in which the concentration polarization is limited by the channel height. The impact of variable transverse velocity on concentration polarization was formulated and evaluated. Simulations of concentration polarization in typical RO membrane channels were presented, and the effects of pressure, membrane
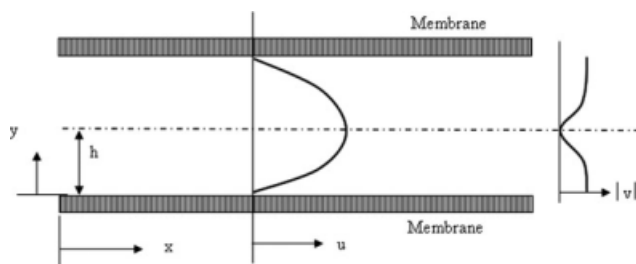
Figure 1. Schematic presentation of longitudinal and transverse flow velocity profiles in the RO membrane channel.

resistance, and channel height on polarization factor were discussed as illustrations of the mechanistic model developed in this article.

Model Development

As indicated earlier, there are two notable differences for concentration polarization in a narrow channel from that in the semiplane over a membrane: (1) the transverse velocity (perpendicular to the membrane surface) in the concentration polarization layer cannot be treated as a constant, and (2) the salt concentration in the bulk solution cannot be treated as unaffected by salt accumulation. The transverse velocity in the membrane channel is first determined for the fully developed parabolic longitudinal flow in the channel. Equations for concentration polarization bounded in the narrow channel and under variable transverse velocity are then developed.

Transverse velocity in a narrow RO channel

The flow velocity profiles in a membrane channel are schematically presented in Figure 1. Mass balance principle on water requires that the transverse velocity, v, in the RO channel decreases from the membrane surface (permeate velocity) to zero at the central line of the feed channel. The magnitude of the transverse velocity, $|v|$, across the membrane channel is schematically depicted on the right side of Figure 1. Only the lower half of a membrane leaf is considered in the following model development because of the asymmetry of the channel. The distance from the membrane surface to the central line will be referred as channel height in this paper. In the following discussions, the x-axis is taken along the longitudinal flow direction with the origin at the channel entrance and the y-axis starts from the membrane surface toward the central line of the channel. A fully developed parabolic longitudinal flow in the membrane channel can be assumed, i.e., the profile of longitudinal flow velocity at any position x in the channel is given by

$$u(x,y) = \frac{3}{2} u_a(x) \left[1 - \left(1 - \frac{y}{h} \right)^2 \right] \qquad (1)$$

where u and u_a are the longitudinal velocity and its average value across the channel height at position x, and h is the distance from membrane surface to the central line of the channel. The average longitudinal velocity is a function of x. The coordinates x and y will not be written out again for velocities to simplify the notations.

The curve on the right hand side of Figure 1 indicates the absolute value of the transverse flow velocity, which increases from zero on the central line to the permeate velocity on the membrane surfaces. The transverse flow velocity in the channel can be accurately determined by combining the continuity equation with Eq. 1 as:

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -\frac{3}{2} \frac{\partial u_a}{\partial x} \left[1 - \left(1 - \frac{y}{h} \right)^2 \right] \qquad (2)$$

where v is the transverse velocity. The transverse velocity vanishes at the central line of the channel, which is described with the boundary condition:

$$v = 0 \quad \text{at} \quad y = h. \qquad (3)$$

Integrations of Eq. 2 with boundary condition Eq. 3 results in

$$v = \frac{3}{2} v_w \left[1 - \frac{y}{h} - \frac{1}{3} \left(1 - \frac{y}{h} \right)^3 \right] \qquad (4)$$

where v_w is the transverse velocity on the membrane surface (permeate velocity)

$$v_w = -h \frac{du_a}{dx}. \qquad (5)$$

The dimensionless transverse velocity, v/v_w, in the RO channel is presented in Figure 2. The transverse flow velocity is seen to keep almost constant in about 5% of the channel height above the membrane surface and then decreases steadily to zero at the central line of the channel. It would be reasonably justifiable to use a constant transverse velocity
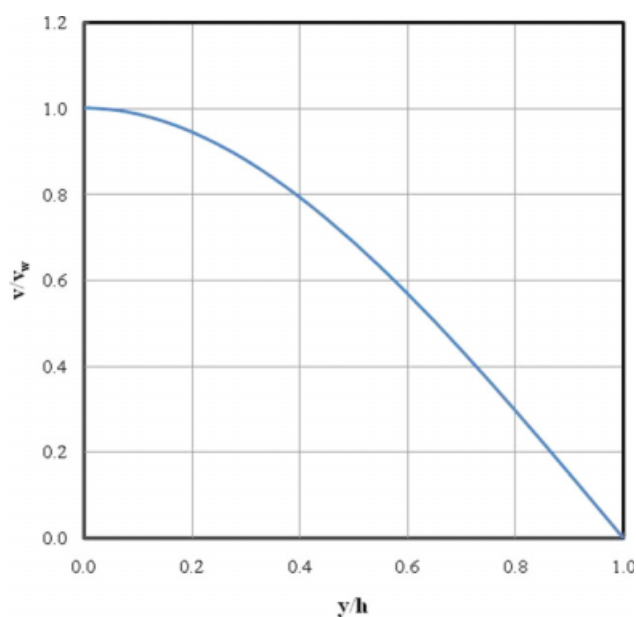

Figure 2. The dimensionless transverse flow velocity profile across the membrane channel height with permeate velocity and fully developed parabolic longitudinal flow.

in the concentration polarization layer if the concentration boundary layer was within 5% of the channel height.

Wall concentration in a narrow membrane channel

Concentration polarization is caused by the accumulation of the retained salt near the RO membrane. The transverse distribution of the retained salt concentration at any distance x from the entrance in the channel is governed[1-2] by

$$vC + D\frac{dC}{dy} = 0 \tag{6}$$

where D is the salt diffusion coefficient, and C is the concentration of retained salt defined as:

$$C = c - c_0 \tag{7}$$

where c and c_0 are the actual salt concentration and feed salt concentration, respectively. The total amount of salt accumulated at distance x in the channel can be written as:

$$\int_0^h C\,dy = C_a h \tag{8}$$

where C_a is the average concentration of retained salt across the channel height.

If the transverse velocity is assumed to be a constant and equal to the permeate velocity, by solving Eqs. 6 and 8, the wall concentration of the retained salt is determined as

$$C'_w = \frac{v_w h}{D} - \frac{C_a}{1 - e^{-\frac{v_w h}{D}}}. \tag{9}$$

For the variable transverse velocity, substituting Eq. 4 into Eq. 6, we have

$$\frac{3}{2}v_w\left[1\frac{y}{h} - \frac{1}{3}\left(1 - \frac{y}{h}\right)^3\right]C + D\frac{dC}{dy} = 0. \tag{10}$$

The wall concentration of the retained salt with the variable transverse velocity can be determined by integration of Eq. 10 with mass conservation condition Eq. 8

$$C_w = \frac{C_a}{\int_0^1 \exp\left\{\frac{3}{2}\frac{v_w h}{D}\left[\frac{1}{12} - y' + \frac{1}{2}(y')^2 - \frac{1}{12}(1 - y')^4\right]\right\}dy'} \tag{11}$$

where $y' = y/h$ is the dimensionless distance from the membrane surface.

By introducing a correcting factor defined as

$$f = \frac{C_w}{C'_w} = \frac{\frac{1}{Pe}[1 - \exp(-Pe)]}{\int_0^1 \exp\left\{\frac{3}{2}Pe\left[\frac{1}{12} - y' + \frac{1}{2}(y')^2 - \frac{1}{12}(1 - y')^4\right]\right\}dy'} \tag{12}$$

where

$$Pe = \frac{v_w h}{D}. \tag{13}$$

The wall concentration of the retained salt in the narrow channel can be expressed as

$$C_w = C_a \frac{v_w h}{D}\frac{f}{\left(1 - e^{-\frac{v_w h}{D}}\right)}. \tag{14}$$

Replacing the retained salt concentration in Eq. 14 by the actual salt concentration results in

$$c_w = c_0 + (c_a - c_0)\frac{v_w h}{D}\frac{f}{1 - e^{-\frac{v_w h}{D}}}. \tag{15}$$

Equation 15 shows that the wall concentration at any point x in a channel of height h is determined by the accumulation of the retained salt $(c_a - c_0)\,h$ and permeate velocity v_w. The salt diffusion coefficient is a fixed value for a given salt. The contribution from the correcting factor f to wall concentration will be discussed in more details later.

Development of concentration polarization in a long membrane channel

The average salt concentration and the average longitudinal velocity change along the membrane channel because of water permeation through the membrane. Therefore, the wall salt concentration, as well as the permeate velocity, will also change along the membrane channel. Applying mass conservation principle to salt, the average salt concentration in a channel with RO membranes of perfect salt rejection is governed by

$$\frac{dc_a}{dx} = \frac{v_w}{u_a h}c_a. \tag{16}$$

By rewriting Eq. 5, the average longitudinal velocity is governed by

$$\frac{du_a}{dx} = -\frac{v_w}{h}. \tag{17}$$

Equations 16 and 17 can be solved numerically by the forward finite difference method along the channel. For the application of the finite difference method, the long channel was divided into n equal segments with the step length $\Delta x = L/n$, where L is the channel length. When the feed salt concentration, feed velocity, permeate velocity are given for entrance section, the average salt concentration, average longitudinal velocity, permeate velocity, and the wall salt concentration can be determined progressively for the entire channel with the finite difference equations below:

$$c_{ai+1} = \left(1 + \frac{v_{wi}}{u_{ai}}\frac{\Delta x}{h}\right)c_{ai} \tag{18}$$

$$u_{ai+1} = u_{ai} - \frac{\Delta x}{h}v_{wi} \tag{19}$$

$$v_{wi+1} = \frac{\Delta P - f_{os}c_{wi}}{R_m} \tag{20}$$
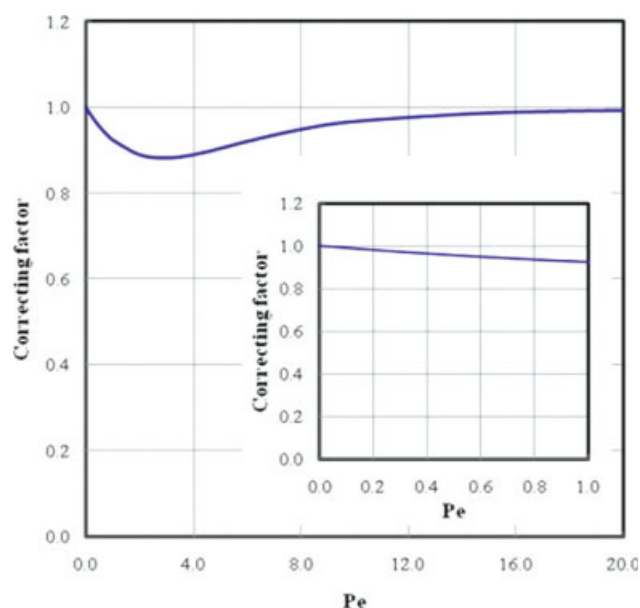
Figure 3. Correcting factor to concentration polarization accounting for the variable transverse velocity at different Peclet numbers.

The small graph shows almost linear decline of the correcting factor from 1.0 to 0.925 for Peclet number from 0.0 to 1.0.

$$c_{wi+1} = c_0 + (c_{ai} - c_0) \frac{f v_{wi} h}{D \left(1 - e^{-\frac{v_{wi} h}{D}}\right)} \tag{21}$$

where ΔP is the driving pressure, and f_{os} is the osmotic pressure coefficient of the salt. Equations 18 and 19 are the finite difference form of Eqs. 16 and 17. To be consistent, the membrane water transport equation and Eq. 15 are also written in the forward difference format in Eqs. 20 and 21. A constant driving pressure is assumed along the membrane channel. Otherwise, the variation in the driving pressure along the channel can be readily tracked by adding another equation.

Discussion

Correcting factor for variable transverse velocity

The correcting factor for a wide range of Peclet number was numerically determined with Eq. 12 and presented in Figure 3. The numerical values of the correcting factor are given in Table 1. The curve shows that there is a global minimum of 0.88 in the correcting factor, which means that the wall salt concentration under the variable transverse velocity at most is about 12% smaller than that determined with the assumption of constant transverse velocity. Peclet numbers of the RO channels that are currently being used in water desalination plants are usually smaller than 1. As shown by the small graph in the figure, the correcting factor decreases linearly from 1 to 0.925 when the Peclet number changes from 0 to 1. Because of the relatively small error, the assumption of constant transverse velocity ($f = 1$) will be used in the following simulations and discussions.

Bounded concentration polarization in the narrow channel

Figure 4 shows the values of polarization factor c_w/c_a as functions of permeate velocities for different channel heights.

Table 1. Calculation of the Correcting Factor

Peclet Number	Numerator of Eq. 12	Denominator of Eq. 12	Correcting Factor
0.0	1.000	1.000	1.000
0.1	0.952	0.961	0.990
0.2	0.906	0.924	0.981
0.3	0.864	0.889	0.972
0.4	0.824	0.855	0.964
0.5	0.787	0.823	0.956
0.6	0.752	0.793	0.949
0.8	0.689	0.736	0.936
1.0	0.633	0.685	0.925
2.0	0.435	0.489	0.890
3.0	0.321	0.364	0.883
4.0	0.251	0.282	0.890
5.0	0.205	0.226	0.905
6.0	0.173	0.188	0.921
8.0	0.133	0.140	0.950
10.0	0.108	0.112	0.968
15.0	0.076	0.077	0.988
20.0	0.059	0.060	0.994

The same amount of retained salt ($c_a - c_0$) h was used for all the channel heights. The retained salt causes 50% increase of the average salt concentration ($c_a = 1.5\ c_0$) in the channel of height 0.2 mm. The broken line on the bottom indicates the case of unbounded concentration polarization, which means the thickness of concentration polarization layer is much smaller than the channel height. The figure shows that concentration polarization is unbounded in the 2 mm channel when the permeate velocity is greater than 4.0×10^{-6} m/s. The wall salt concentrations for the channel heights smaller than 1.0 mm are notably higher than the unbounded case. The higher wall concentrations in the narrower channels can be
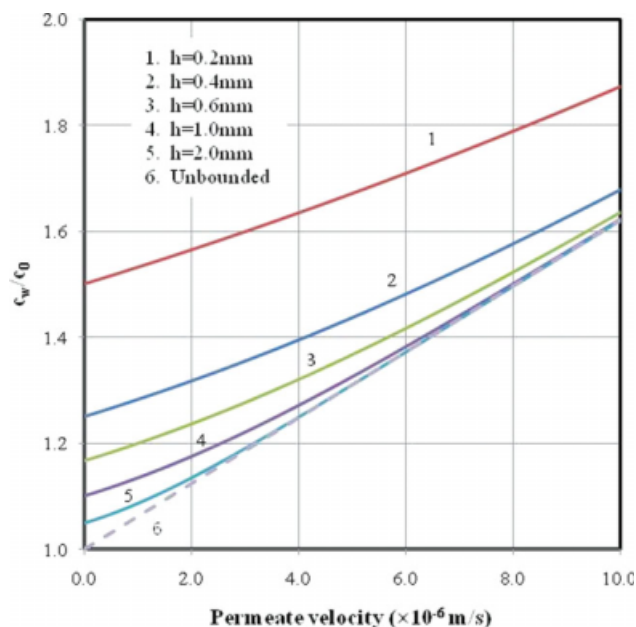

Figure 4. The impact of channel height on wall salt concentration under various permeate velocities.

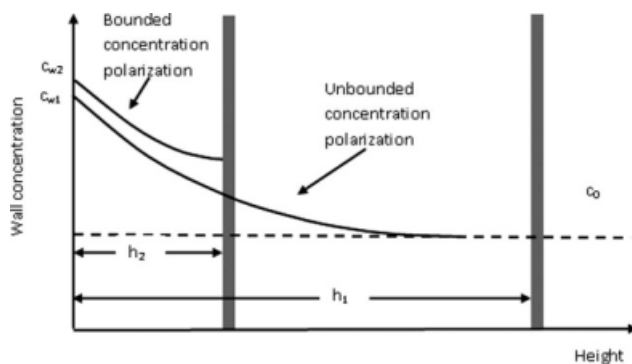The broken line is for the unbounded concentration polarization.

Figure 5. Schematic for the bounded and unbounded polarization layers in channel of different heights.

Parameter	Unit	Value
Membrane resistance	Pa s/m	3.00×10^{11}
Channel height	m	3.50×10^{-4}
Channel length	m	8
Feed salt concentration	mg/L	35,000
Osmotic pressure coefficient	Pa/(mg/L)	73.9
Feed cross flow velocity	m/s	0.1
Pressure	Pa	6.0×10^{6}
Diffusion coefficient	m^2/s	1.61×10^{-9}
Correcting factor		1

schematically explained in Figure 5. The restriction of the salt to expand to a wider range by the channel height h_2 results in a higher wall concentration c_{w2} than c_{w1} in the unbounded case for the channel height h_1.

The restriction of channel height on the development of concentration polarization was studied by Newman in 1960s when he was discussing the applicability of Leveque equation to a channel. He indicated that the boundary layer theory could be safely used[8,14] (that means the concentration polarization layer is smaller than the channel height) when

$$\frac{x}{h} < 0.01 N_{Re} S_c \qquad (22)$$

where x is the distance from the entrance, N_{Re} ($= u_a h/v$ with v being kinetic viscosity) is the longitudinal Reynolds number, and Sc ($= v/D$) is the Schmidt number.

The typical values for the height[15–17] and feed velocity[11] of RO channels are 0.71 mm and 0.1 m/s, respectively. The diffusion coefficient of sodium chloride[9] is 1.61×10^{-9} m^2/s. The longitudinal Reynolds number and Schmidt number determined from these data are 71 and 621, respectively. According to Eq. 22, concentration polarization develops unrestrictedly only in a short distance of about 31 cm from the entrance of the RO channel.

Features of concentration polarization in a long RO channel

The average and wall salt concentrations were calculated along a membrane channel of 8-m long with finite difference model, in which 100 steps were used for the channel length. Typical parameters for seawater desalinations in Table 2 will be used in all simulations of the long RO channel unless other specified. It is worthy to point out that spacers in the RO channel cannot be explicitly considered in the current model. However, its impact on concentration polarization can be included by replacing the salt diffusion coefficient with the dispersion coefficient of the spacer-filled channel.[12] The results are presented in Figure 6. It can be seen that both wall concentration and average concentration increase downstream along the channel. The wall concentration increases at a higher rate in a front portion of the channel and a lower rate in the rest of the channel. This behavior is

more clearly shown by the polarization factor c_w/c_a, which increases from 1 to the maximum point at a place slightly beyond 1 m in the channel then decreases gradually along the rest of the channel.

The impacts of driving pressure on polarization factor c_w/c_a were calculated for a typical RO channel for seawater desalination and the results are presented in Figure 7. The results show that concentration polarization is very sensitive to the driving pressure. The peak concentration factor increases from 1.06 at the pressure of 40 bars to 1.46 at the pressure of 80 bars. Another interesting finding from the simulations is that the peak polarization factors occur in the first 2 m of the channels. This phenomenon demonstrates the important roles that both salt accumulation and permeate velocity play on concentration polarization. The initial increase in the polarization factor is mainly due to the accumulation of the retained salt. After the peak value, polarization factors decline because the permeate velocities decrease substantially downstream along the channel. The decline of permeate velocities can also be indirectly attributed to salt accumulation that reduces the net driving pressure of the RO filtration. The pressure loss along the seawater RO channel is relatively small, which is only about 5–10% of the feed driving pressure.[18]
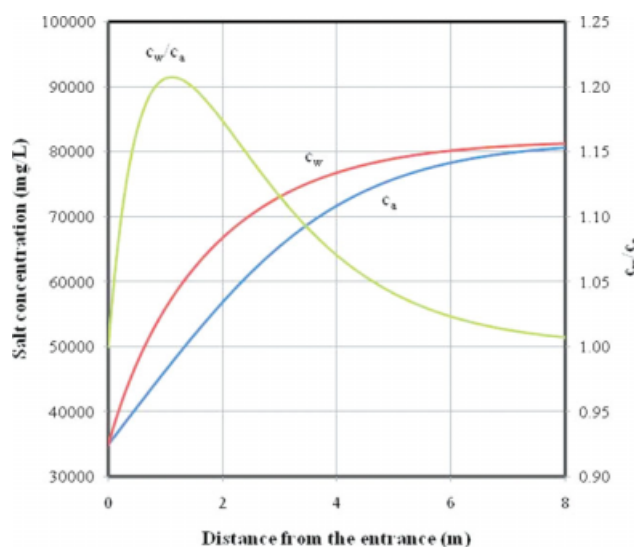

Figure 6. Average salt concentration, wall concentration, and polarization factor along a typical seawater desalination RO channel.
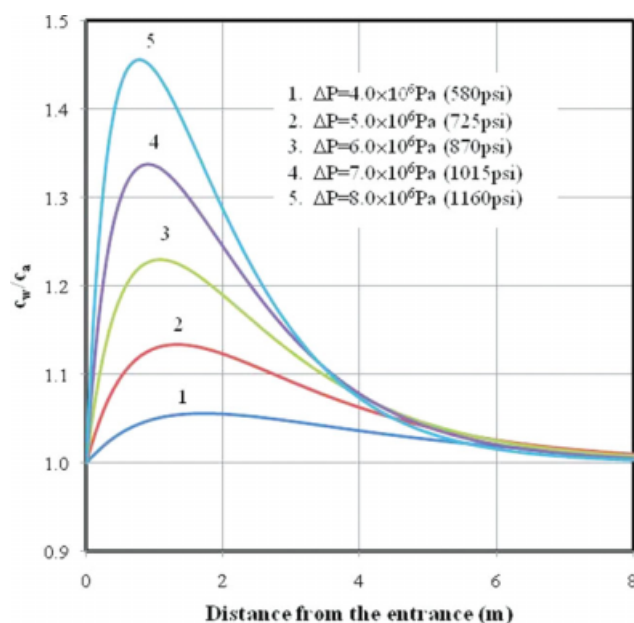
Figure 7. Polarization factor along a typical seawater desalination RO channel under various applied pressures.

Figure 9. Polarization factor along a typical seawater desalination RO channel under various channel heights.

Concentration polarization in a long RO channel is also simulated for various membrane resistance and the results are presented in Figure 8. The peak concentration factor is found to shift downstream with decreasing peak value for increasing membrane resistance. It implies that concentration polarization might not be substantially developed for the RO membranes of old generation that was characterized by resistance of about one order in magnitude higher than the current ones. It can be explained by the slower accumulation of
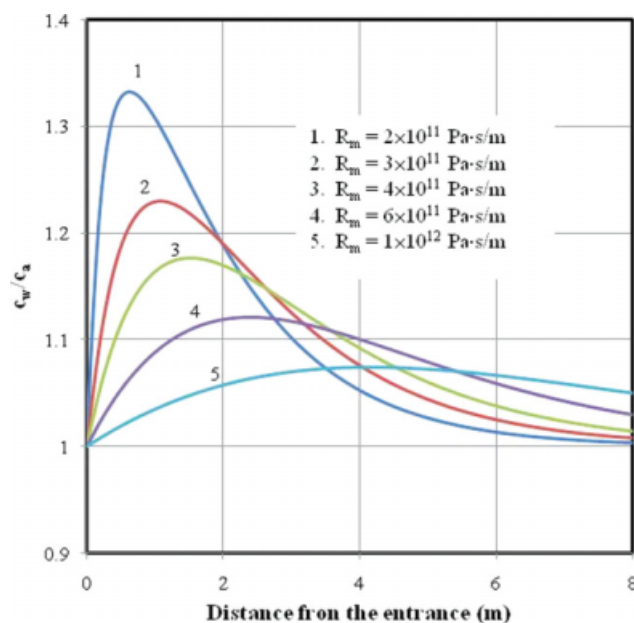

Figure 8. Polarization factor along a typical seawater desalination RO channel under various membrane resistances.
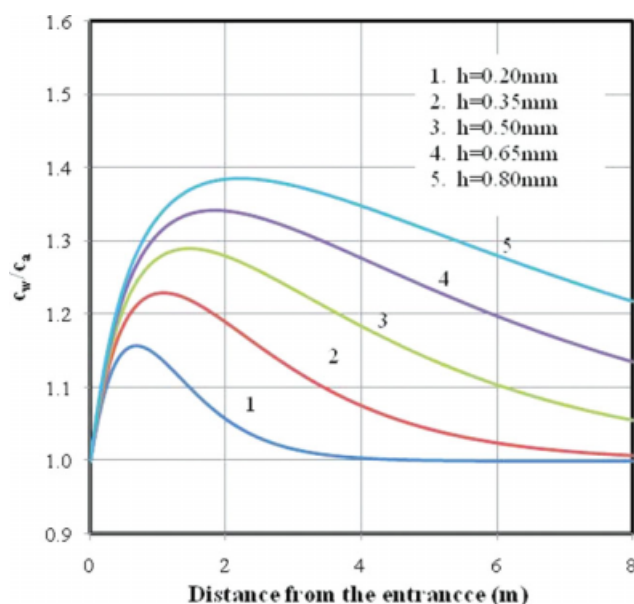
the retained salt and smaller permeate velocity with the old generation of RO membranes. Unlike those in Figure 7, substantial differences in the polarization factors are found in the tail part of the channels for different membrane resistances in Figure 8. The polarization factors appear in the opposite order in the tail part to those in the front part of the channels. For instance, the polarization factor for membrane resistance of 2×10^{11} Pa s/m is the largest in the front part but the smallest in the tail part of the channel.

The impact of channel height on the polarization factor is simulated and shown in Figure 9. It can be seen that the polarization factors increase with the channel height. The reason is that there is a large room for concentration to develop in the channel of greater height. As described schematically in Figure 5, concentration polarization layer can develop across the whole channel height sooner in a narrow channel. The rapid increase in the average salt concentration limits the further increase in polarization factor (c_w/c_a). Another interesting point is that he polarization factor for larger channel height decreases more slowly along the channel after it reaches the maximum. This behavior indicates higher permeate velocity remains in the rest of the channel for larger channel height. It implies that the same membrane area in the channel of larger height will produce more water.

Conclusions

Concentration polarization in a narrow RO membrane channel is mechanistically modeled using the concept of retained salt. Because of the small channel height, the concentration polarization may develop across the entire channel height and under variable transverse velocity. It is demonstrated in this article that the wall salt concentrations in the typical RO channels would be substantially underestimated if concentration polarization was treated to develop in an

unbounded channel. However, the impact of variable transverse velocity on the wall salt concentrations is proven to be relatively small. The relative simple equations developed under the assumption of constant transverse velocity can be satisfactorily used to evaluate concentration polarization in the narrow RO channel with an error within a few percentages for membrane channels with Peclet number smaller than 1. Simulations of concentration polarization in typical RO channels show that the polarization factor (c_w/c_a) usually increases rapidly to a maximum value in a short distance from the entrance and decreases gradually in the rest of the membrane channels. The model developed in this article is demonstrated to have the ability to reveal more detailed insights on concentration polarization in the narrow RO channels that cannot be done before.

Notation

c = salt concentration (mg/L)
C = retained salt concentration (mg/L)
c_a = average salt concentration (mg/L)
C_a = average retained salt concentration (mg/L)
c_w = wall salt concentration (mg/L)
C_w = wall retained salt concentration (mg/L)
C_w' = wall retained salt concentration for constant transverse velocity (mg/L)
c_0 = feed salt concentration (mg/L)
D = salt diffusion coefficient (m²/s)
Δx = step length in the finite difference model of the RO channel
f_{os} = osmotic pressure coefficient (Pa/(mg/L))
f = correcting factor of wall concentration for variable transverse velocity
h = half height of the RO channel (m)
L = length of the RO channel (m)
n = number of steps in the finite difference model of the RO channel
N_{Re} = longitudinal Reynolds number
Pe = Peclet number defined as vh/D
Sc = Schmidt number.
u_a = average longitudinal flow velocity (m/s)
v = transverse flow velocity (m/s)
v_w = transverse flow velocity on the membrane surface (permeate velocity) (m/s)
y' = dimensionless distance from the membrane surface.

Greek letters

ΔP = driving pressure (Pa)
$\Delta \pi$ = osmotic pressure (Pa)
v = kinetic viscosity (m²/s)

Subscripts

i = ith segment in the difference model of RO channel

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